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Álgebra

Tema: Números complejos II

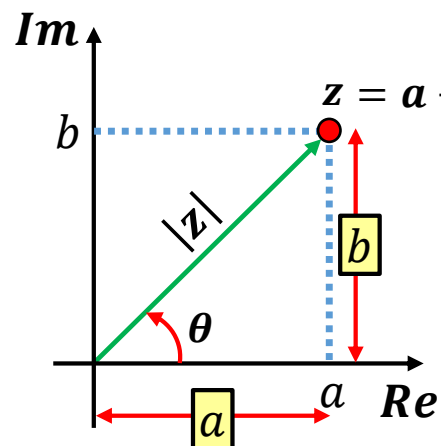
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FORMA POLAR Y EXPONENCIAL DE UN NÚMERO COMPLEJO

Sea $z = a + bi \in \mathbb{C}$ tal que $a > 0 \wedge b > 0$

Graficamos z en el **PLANO COMPLEJO**.



$$\operatorname{sen} \theta = \frac{b}{|z|}$$

$$b = |z| \operatorname{sen} \theta$$

$$\cos \theta = \frac{a}{|z|}$$

$$a = |z| \cos \theta$$

Como $z = a + bi$

$$\Rightarrow z = |z| \cos \theta + |z| \operatorname{sen} \theta \cdot i$$

$$\boxed{z = |z|(\cos \theta + i \operatorname{sen} \theta)} \quad \text{Forma polar}$$

θ : Argumento de z (**Arg(z)**) $0 \leq \theta < 2\pi$

Observación

$$\cos \theta + i \operatorname{sen} \theta = \operatorname{cis} \theta = e^{i\theta}$$

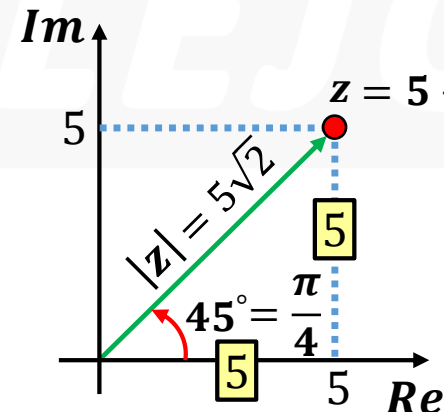
$$\boxed{z = |z| \operatorname{cis}(\theta)} \quad \text{Forma cis}$$

$$\boxed{z = |z| e^{i\theta}} \quad \text{Forma exponencial}$$

Ejemplos

Expresa en las otras formas los siguientes números:

① $z = 5 + 5i$



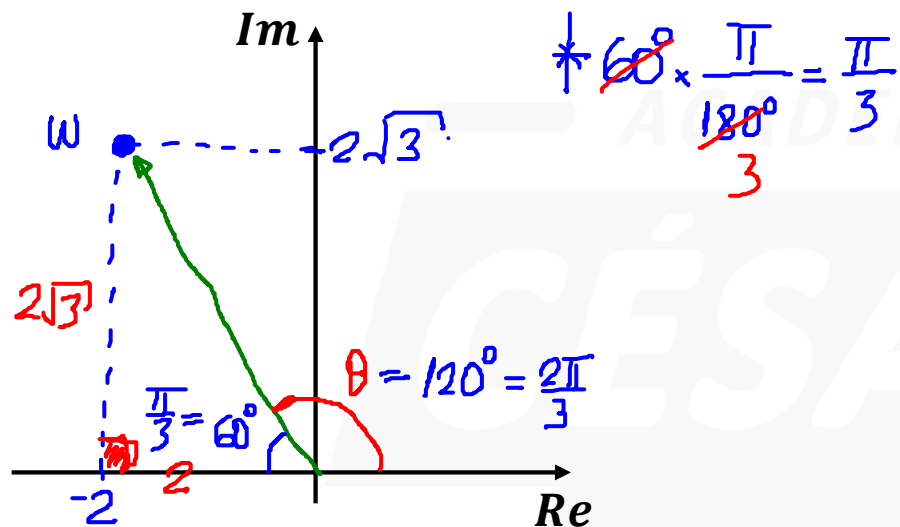
$$z = 5\sqrt{2} \left(\cos \frac{\pi}{4} + i \operatorname{sen} \frac{\pi}{4} \right)$$

$$z = 5\sqrt{2} \operatorname{cis} \frac{\pi}{4}$$

$$z = 5\sqrt{2} e^{\frac{\pi}{4}i}$$

② $w = -2 + 2\sqrt{3}i$

$|w| = 4$



Juego:

$$w = 4 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$w = 4 \operatorname{cis} \frac{2\pi}{3}$$

$$w = 4 e^{\frac{2\pi}{3}i}$$

COMPLEJOS NOTABLES

$$1 = (\cos 0 + i \sin 0) = \operatorname{cis}(0) = e^{0i}$$

$$-1 = (\cos \pi + i \sin \pi) = \operatorname{cis}(\pi) = e^{\pi i}$$

$$i = \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) = \operatorname{cis} \left(\frac{\pi}{2} \right) = e^{\frac{\pi}{2}i}$$

$$-i = \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right) = \operatorname{cis} \left(\frac{3\pi}{2} \right) = e^{\frac{3\pi}{2}i}$$

TEOREMAS

Sean los números complejos z y w no nulos.

$$z = |z|(\cos\theta + i\operatorname{sen}\theta) = |z|\operatorname{cis}(\theta) = |z|e^{i\theta}$$

$$w = |w|(\cos\alpha + i\operatorname{sen}\alpha) = |w|\operatorname{cis}(\alpha) = |w|e^{i\alpha}$$

Donde se cumple:

$$z \cdot w = \begin{cases} |z| \cdot |w| [\cos(\theta + \alpha) + i\operatorname{sen}(\theta + \alpha)] \\ |z| \cdot |w| \operatorname{cis}(\theta + \alpha) \\ |z| \cdot |w| e^{(\theta + \alpha)i} \end{cases}$$

$$\frac{z}{w} = \begin{cases} \frac{|z|}{|w|} [\cos(\theta - \alpha) + i\operatorname{sen}(\theta - \alpha)] \\ \frac{|z|}{|w|} \operatorname{cis}(\theta - \alpha) \\ \frac{|z|}{|w|} e^{(\theta - \alpha)i} \end{cases}$$

Ejemplos

Sean los números complejos

$$z = 6 \left(\cos \frac{\pi}{3} + i \operatorname{sen} \frac{\pi}{3} \right) \quad \text{y} \quad w = 3 \left(\cos \frac{\pi}{4} + i \operatorname{sen} \frac{\pi}{4} \right)$$

$$z = 6 \cdot e^{\frac{\pi}{3}i}$$

$$w = 3e^{\frac{\pi}{4}i}$$

$$z \cdot w = (6 \cdot e^{\frac{\pi}{3}i}) (3e^{\frac{\pi}{4}i})$$

$$= 18 e^{(\frac{\pi}{3} + \frac{\pi}{4})i} = 18 e^{\frac{7\pi}{12}i} = 18 \operatorname{cis} \left(\frac{7\pi}{12} \right)$$

$$= 18 \left(\cos \frac{7\pi}{12} + i \operatorname{sen} \frac{7\pi}{12} \right)$$

$$\frac{z}{w} = \frac{6 \cdot e^{\frac{\pi}{3}i}}{3 e^{\frac{\pi}{4}i}} = 2 e^{(\frac{\pi}{3} - \frac{\pi}{4})i} = 2 e^{\frac{\pi}{12}i}$$

$$= 2 \operatorname{cis} \left(\frac{\pi}{12} \right) = 2 \left(\cos \frac{\pi}{12} + i \operatorname{sen} \frac{\pi}{12} \right)$$

TEOREMA DE MOIVRE

Dado el número complejo no nulo

$$z = |z|(\cos\theta + i\operatorname{sen}\theta)$$

Se tiene ($n \in \mathbb{N}$)

$$I. \quad z^n = \begin{cases} |z|^n [\cos(\theta n) + i\operatorname{sen}(\theta n)] \\ |z|^n \operatorname{cis}(\theta n) \\ |z|^n e^{(\theta n)i} \end{cases}$$

Ejemplos

$$\begin{aligned} 1. (\cos 36^\circ + i\operatorname{sen} 36^\circ)^5 &= \cos(36^\circ \cdot 5) + i\operatorname{sen}(36^\circ \cdot 5) \\ &= \cos(180^\circ) + i\operatorname{sen}(180^\circ) \end{aligned}$$

$$= -1 + i \cdot 0$$

$$= -1$$

$$R + (2\pi + \theta) = R + \theta$$

$$R + (\cancel{\theta} + 2\pi) = R + 2\pi$$

$$2. \left(\sqrt{2} \operatorname{cis} \frac{\pi}{4}\right)^{12} = \cancel{\sqrt{2}}^2 \cdot \operatorname{cis} \left(\cancel{12} \cdot \frac{\pi}{4}\right)$$

$$= 2^6 \operatorname{cis}(3\pi) = 64 (\underbrace{\cos 3\pi}_{-1} + i \underbrace{\operatorname{sen} 3\pi}_0)$$

$$= -64$$

$$3. \left(4e^{\frac{5\pi}{3}i}\right)^6 = 4^6 \cdot e^{\frac{5\pi}{3} \cdot 6i} = 4^6 \cdot e^{10\pi i}$$

$$= 4096 \cdot \left(\underbrace{\cos 10\pi}_{\underbrace{\cos 2\pi}_1} + i \underbrace{\operatorname{sen} 10\pi}_{\underbrace{\operatorname{sen} 2\pi}_0} \right)$$

$$= 4096$$

Aplicación

Reducir el siguiente complejo:

$$Z = \frac{(Cis\ 12^\circ)^4 \cdot (5\ Cis\ 16^\circ)^5}{(5\ Cis\ 7^\circ)^3 \cdot (Cis\ 35^\circ)^2}$$

indique $Re(Z) + Im(Z)$

- A) 26 ~~B) 35~~ C) 44 D) 68 E) 76

Resolución

$$Z = \frac{(e^{12^\circ i})^4 \cdot 5^5 (e^{16^\circ i})^5}{5^3 (e^{7^\circ i})^3 \cdot (e^{35^\circ i})^2}$$

$$Z = \frac{5^5 e^{48i} \cdot e^{80i}}{5^3 e^{21i} \cdot e^{70i}}$$

$$Z = \frac{5^2 e^{128i}}{e^{91i}}$$

$$Z = 25 \cdot e^{37^\circ i}$$

$$Z = 25 \left(\underbrace{\cos 37^\circ}_{\frac{4}{5}} + i \underbrace{\sin 37^\circ}_{\frac{3}{5}} \right)$$

$$Z = 20 + 15i$$

$$\begin{aligned} \text{Por } Re(Z) + Im(Z) &= 20 + 15 \\ &= 35 \end{aligned}$$

II.

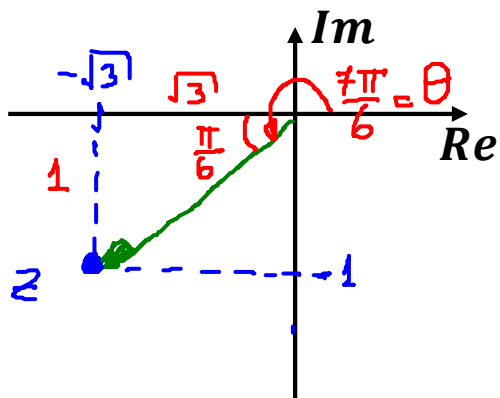
$${}^n\sqrt{z} = \begin{cases} {}^n\sqrt{|z|} \left(\cos\left(\frac{\theta + 2k\pi}{n}\right) + i \operatorname{sen}\left(\frac{\theta + 2k\pi}{n}\right) \right) \\ {}^n\sqrt{|z|} \operatorname{cis}\left(\frac{\theta + 2k\pi}{n}\right) \\ {}^n\sqrt{|z|} e^{\left(\frac{\theta + 2k\pi}{n}\right)i} \end{cases}$$

donde $k = 0, 1, 2, \dots, n - 1$.

Aplicación

Si $z = -\sqrt{3} - i$, calcule $\sqrt[4]{z}$.

Resolución



$$n = 4$$

$$|z| = 2$$

$$\sqrt[4]{z} = \sqrt[4]{-\sqrt{3} - i} = \sqrt[4]{2} \operatorname{cis}\left(\frac{\frac{7\pi}{6} + 2k\pi}{4}\right)$$

Juego: $k = 0; 1; 2; 3$

$$\sqrt[4]{z} = \begin{cases} k=0 : \omega_0 = \sqrt[4]{2} \operatorname{cis}\left(\frac{7\pi}{24}\right) \\ k=1 : \omega_1 = \sqrt[4]{2} \operatorname{cis}\left(\frac{19\pi}{24}\right) \\ k=2 : \omega_2 = \sqrt[4]{2} \operatorname{cis}\left(\frac{31\pi}{24}\right) \\ k=3 : \omega_3 = \sqrt[4]{2} \operatorname{cis}\left(\frac{43\pi}{24}\right) \end{cases}$$

+12
+12
+12

$$\sqrt[2]{4} = \pm 2$$

$$\sqrt[4]{4} = 2$$

$$\sqrt[3]{8} = \sqrt[3]{8+0i} = \sqrt[3]{8}$$

$$A) \pm 2 \quad B) 2$$

Aplicación

Calcule el área del polígono que se forma al unir

los afijos de $w = \sqrt[5]{243 \operatorname{cis} \left(\frac{3\pi}{4} \right)} = \sqrt[5]{z}$

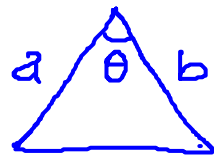
Resolución

$$k: 0; 1; 2; 3; 4.$$

$$\sqrt[5]{z} = \sqrt[5]{243} \operatorname{cis} \left(\frac{\frac{3\pi}{4} + 2k\pi}{5} \right)$$

$$\sqrt[5]{z} = \begin{cases} k=0: & w_0 = 3 \operatorname{cis} \left(\frac{3\pi}{20} \right) \\ k=1: & w_1 = 3 \operatorname{cis} \left(\frac{11\pi}{20} \right) \\ k=2: & w_2 = 3 \operatorname{cis} \left(\frac{19\pi}{20} \right) \\ k=3: & w_3 = 3 \operatorname{cis} \left(\frac{27\pi}{20} \right) \\ k=4: & w_4 = 3 \operatorname{cis} \left(\frac{35\pi}{20} \right) \end{cases}$$

Recordar

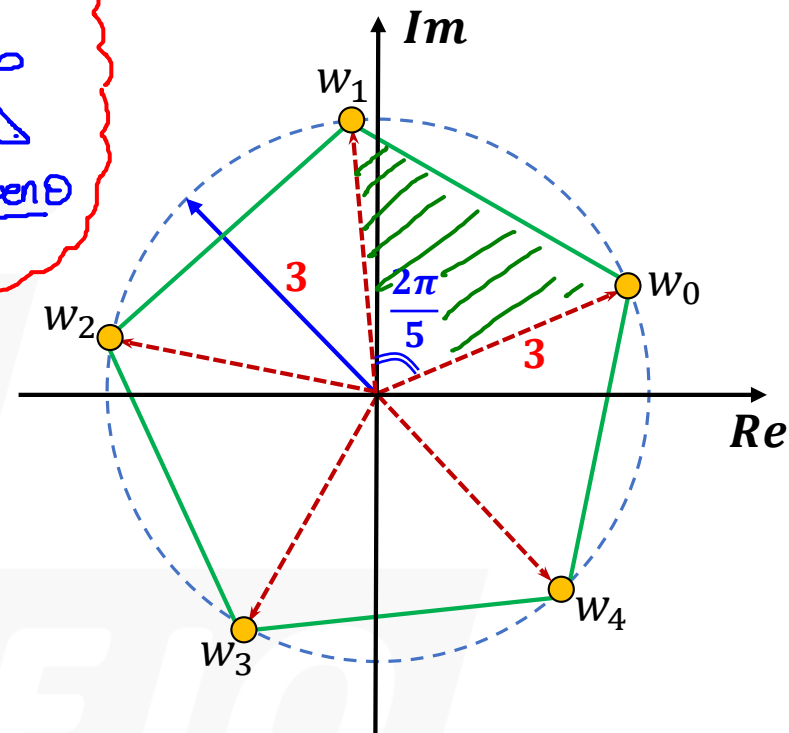


$$A = \frac{ab \sin \theta}{2}$$

$$|z| = 243$$

$$\theta = \frac{3\pi}{4}$$

$$n = 5$$



$$A_{\triangle} = \frac{(3)(3)}{2} \sin \left(\frac{2\pi}{5} \right)$$

$$\text{Por } A_T = 5 \left(\frac{9}{2} \sin \left(\frac{2\pi}{5} \right) \right) = \frac{45}{2} \sin \left(\frac{2\pi}{5} \right) u^2$$

PROPIEDADES

- | | |
|------|--|
| I. | $\text{Arg}(z \cdot w) = \text{Arg}(z) + \text{Arg}(w)$ |
| II. | $\text{Arg}\left(\frac{z}{w}\right) = \text{Arg}(z) - \text{Arg}(w)$ |
| III. | $\text{Arg}(z^n) = n \cdot \text{Arg}(z)$ |

Aplicación

Si $|\bar{z}(1+i)| = 3\sqrt{2}$; $\text{Arg}[z^2 \cdot (\sqrt{3} + i)] = \frac{2\pi}{3}$

entonces el número z en su forma polar es:

Resolución

$$* |\bar{z}(1+i)| = 3\sqrt{2}$$

$$|\bar{z}| |1+i| = 3\sqrt{2}$$

$$|\bar{z}|(\sqrt{2}) = 3\sqrt{2} \rightarrow |z| = 3$$

$$* \text{Arg}(z^2(\sqrt{3}+i)) = \frac{2\pi}{3}$$

$$\text{arg}(z^2) + \text{arg}(\sqrt{3}+i) = \frac{2\pi}{3}$$

$$2\text{arg}(z) + \frac{\pi}{6} = \frac{2\pi}{3}$$

$$2\text{arg}(z) = \frac{2\pi}{3} - \frac{\pi}{6} = \frac{\pi}{2}$$

$$\text{arg}(z) = \frac{\pi}{4} = \theta$$

$$\therefore z = 3 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) = 3 \text{cis} \left(\frac{\pi}{4} \right)$$

RESULTADOS IMPORTANTES

$$|\cos\theta + i\operatorname{sen}\theta| = |\operatorname{cis}\theta| = |e^{i\theta}| = 1$$

$$\cos\theta - i\operatorname{sen}\theta = \cos(-\theta) + i\operatorname{sen}(-\theta) = \operatorname{cis}(-\theta)$$

$$\frac{1}{\cos\theta + i\operatorname{sen}\theta} = \cos\theta - i\operatorname{sen}\theta$$

$$\text{Si } z = |z|\operatorname{cis}(\theta) \Rightarrow \begin{cases} \bar{z} = |z|\operatorname{cis}(-\theta) \\ z^* = |z|\operatorname{cis}(\theta + \pi) \end{cases}$$

$$e^{x+yi} = e^x \operatorname{cis}(y)$$

Aplicaciones

Indique el módulo y argumento de los siguientes números complejos:

$$\diamond \cos 30^\circ - i\operatorname{sen} 30^\circ =$$

$$\diamond \text{Problema 8 (Sugerencia)}$$

$$z_2 = -2e^{\theta i}$$

$$= 2(-1 \cdot e^{\theta i})$$

$$= 2(e^{\pi i} \cdot e^{\theta i})$$

$$\diamond e^{3+4i} =$$

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GRACIAS

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